

ALGEBRA 1

Chapter 10, Section 5

Factoring $x^2 + bx + c$

VOCABULARY:

- None

GOALS:

- To factor a quadratic expression of the form $x^2 + bx + c$.

FACTORIZING $x^2 + bx + c$

You know from the FOIL method that $(x + p)(x + q) = x^2 + (p + q)x + pq$. So to factor $x^2 + bx + c$, you need to find numbers p and q such that

$$p + q = b \text{ and } pq = c$$

Because $x^2 + (p + q)x + pq = x^2 + bx + c$ if and only if $p + q = b$ and $pq = c$

Example: $x^2 + 6x + 8 = (x + 4)(x + 2)$ $4 + 2 = 6$ and $4 \cdot 2 = 8$

1. Factoring when b and c are positive:

Factor $x^2 + 3x + 2$.

For this trinomial, $b = 3$ and $c = 2$. You need to find two numbers whose sum is 3 and whose product is 2.

$$\begin{aligned}x^2 + 3x + 2 &= (x + p)(x + q) && \text{Check for factors of 2 which have a sum of 3. They are 1 \& 2.} \\ &= (x + 1)(x + 2)\end{aligned}$$

So the factors of the trinomial are $(x + 1)(x + 2)$.

2. Factoring when b is negative and c is positive:

Factor $x^2 - 5x + 6$.

For this trinomial, $b = -5$ and $c = 6$. You need to find two numbers whose sum is -5 and whose product is 6 therefore both p and q must be negative numbers.

$$\begin{aligned}x^2 - 5x + 6 &= (x + p)(x + q) && \text{Check for factors of 6 which have a sum of -5. They are -2 \& -3.} \\ &= (x - 2)(x - 3)\end{aligned}$$

So the factors of the trinomial are $(x - 2)(x - 3)$.

3. Factoring when b and c are negative:

Factor $x^2 - 2x - 8$.

For this trinomial, $b = -2$ and $c = -8$. You need to find two numbers whose sum is -2 and whose product is -8 therefore for p and q one must be negative and the other positive.

$$\begin{aligned}x^2 - 2x - 8 &= (x + p)(x + q) && \text{Check for factors of -8 which have a sum of -2. They are -4 \& 2.} \\ &= (x - 4)(x + 2)\end{aligned}$$

So the factors of the trinomial are $(x - 4)(x + 2)$.

4. Factoring when b is positive and c is negative:

Factor $x^2 + 7x - 18$.

For this trinomial, $b = 7$ and $c = -18$. You need to find two numbers whose sum is 7 and whose product is -18 therefore for p and q one must be negative and the other positive.

$$\begin{aligned}x^2 + 7x - 18 &= (x + p)(x + q) \quad \text{Check for factors of -18 which have a sum of 7. They are -2 \& 9.} \\ &= (x - 2)(x + 9)\end{aligned}$$

So the factors of the trinomial are $(x - 2)(x + 9)$.

It is also important to realize that many trinomials cannot be factored into linear factors with integer coefficients. A quadratic trinomial ($x^2 + bx + c$) can be factored only if the discriminant ($b^2 - 4ac$) is a perfect square.

5. Using the discriminant:

Tell whether this trinomial can be factored: $x^2 + 3x - 6$.

$$\begin{aligned}b^2 - 4ac &= 3^2 - 4(1)(-6) \\ &= 33\end{aligned}$$

The discriminant is not a perfect square, so the trinomial cannot be factored.

Find the factors of the following:

a.) $x^2 + 5x + 6$	b.) $x^2 - 3x + 2$
c.) $x^2 - x - 6$	d.) $x^2 + 5x - 24$
e.) $x^2 + 3x - 18$	f.) $x^2 - 45x - 900$